

# Driver assisted steering system for reversing an articulated vehicle

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**Abstract**—This paper presents the development and implementation of a controller for fully and semi-automated reversing of a truck and trailer system with two pivot joints.

**Index Terms**—truck and trailer reversing, line tracking, kinematics, non-linear control

## I. INTRODUCTION

The use of truck and trailer systems, also known as articulated vehicles has become a vital part of any industry. Choosing articulated vehicles over rigid vehicles have several major advantages. These vehicles can take much sharper turns than similar sized rigid vehicles and it is possible for the wheels to keep in contact with the ground even when the surface is uneven. Since these vehicles can carry more load, cost of transportation can be reduced with a reduced fuel consumption [6], [9].

However, training a driver to control such a vehicle takes a lot of effort and time [2]. Especially when reversing, the trailer moves in an unpredictable direction with respect to the truck movement. Hence the reversing movement is very complex [5]. In certain cases when reversing, there is more tendency for the vehicle to fold into a position it cannot come out without a forward motion. This is called the jackknife effect [1]. Even the professional drivers with a high skill level can fail when it comes to reversing. In this, an articulated vehicle with two pivot points is considered. For such a vehicle reversing is almost impossible without any guidance [4]. Hence it will be beneficial if there is some way to assist the driver by providing the necessary guidelines for accurate reversing. Then the driver can reverse the vehicle along the desired path without worrying about the trailer behavior. Developing such a driver assisted steering system is the ultimate objective of this work. Though driver assisted systems are available for single body vehicles, for multibody vehicles, the technology it is still under development [4]. This paper first presents a controller for such a system with simulation verification. Then the paper presents the experimental validation results of the controller for two cases: fully autonomous small scale prototype, and semi-autonomous (driver assisted) larger scale prototype. The controller uses an onboard vision based localization system.

Published results for multi trailer systems are significantly less compared to single trailer systems. Derivation of the kinematic equations of motion for an N-trailer configuration can be found in [3], [4], [7], while single trailer systems are considered in [5], [8], [9]. The paper [1] proposes a new hybrid feedback control scheme for two trailer system, considering low level controls for backward driving along a line and a curve while [2] presents an approach for stabilization and path tracking for a two trailer configuration using an LQ-controller that stabilizes the internal angles and a pure pursuit controller is used for path tracking. The work presented in [3] solves the motion planning problem for an N-trailer system by converting the system into chained form. A driver assistance system for backward docking of an Ntrailer vehicle is discussed in [4], [7], using a cascaded VFO control law as a control assistance function while [6] proposes a novel approach to control multiple trailers connected to the front bumper of a truck.

To the best of our knowledge, a compact on-board vision based solution for a two trailer reversing problem is not available. Most of the controllers depend on external vision based localization techniques [4], [7] while some processing is done outside the system [2].

The outline of the paper is as follows. In section II the mathematical model and derivation of the controller to stabilize the system is discussed. In section III simulation results of the controller are presented and finally in section IV verification of the controller using two hardware prototypes is presented.

## II. MATHEMATICAL MODEL OF THE SYSTEM

### A. Notation

Let's consider a system with two trailers and the engine part of the truck as shown in Fig.1. Earth frame is represented by  $\mathbf{e}$  and  $R_z$  is the rotational matrix around axis 3.

$$\mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \quad R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$X(L)$  represents a translation  $L$  along the axis 1.

$$X(L) = \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix}$$

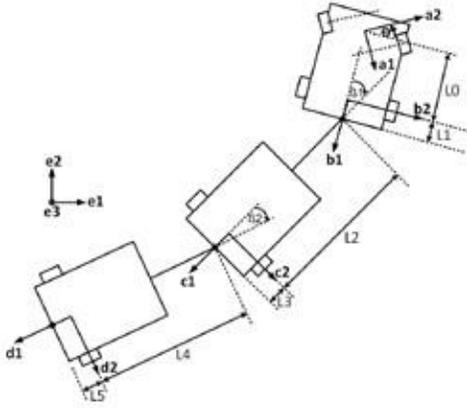


Fig. 1. Reference frames for the trailer system analysis.

### B. Coordinate system

The equations are written with respect to earth frame.

Coordinates of each frame with respect to earth frame can be written as, where F is defined in the appendix. Using small angle approximation the linearized version of the above equations are:

$$ob = oa + Rz(\alpha - \theta)X(L_0)$$

$$oc = oa + Rz(\alpha - \theta)X(L_0 + L_1) + Rz(\alpha - \theta - \beta_1)X(L_2)$$

$$\begin{bmatrix} \dot{\beta}_1 \\ \dot{\beta}_2 \end{bmatrix} = V_{a1} \begin{bmatrix} \frac{1}{L_2} & 0 \\ -\frac{(L_3+L_4)}{L_2L_4} & \frac{1}{L_4} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + V_{a1} \sin(\theta) \begin{bmatrix} -\frac{(L_1+L_2)}{L_0L_2} \\ \frac{L_1(L_3+L_4)}{L_0L_2L_4} \end{bmatrix}$$

$$od = oa + Rz(\alpha - \theta)X(L_0 + L_1) + Rz(\alpha - \theta - \beta_1)X(L_2 + L_3) + Rz(\alpha - \theta - \beta_1 - \beta_2)X(L_4)$$

Where  $oa$  is the coordinates of  $a$  frame with respect to  $e$ . Then the velocities of the origins of each frame with respect to earth frame can be written as,

$$\begin{bmatrix} \dot{o}_a \\ \dot{o}_b \\ \dot{o}_c \\ \dot{o}_d \end{bmatrix} = V_{a1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{o}_a = Rz(\alpha)\dot{\theta}$$

$$\dot{o}_b = \dot{o}_a + Rz(\alpha - \theta)\Omega_z(\alpha' - \theta')X(L_0)$$

$$\dot{o}_c = \dot{o}_a + Rz(\alpha - \theta)\Omega_z(\alpha' - \theta')X(L_0 + L_1) + Rz(\alpha - \theta - \beta_1)\Omega_z(\alpha' - \theta' - \beta_1')$$

$$\dot{o}_d = \dot{o}_a + Rz(\alpha - \theta)\Omega_z(\alpha' - \theta')X(L_0 + L_1) + Rz(\alpha - \theta - \beta_1)\Omega_z(\alpha' - \theta' - \beta_1')$$

$$\dot{o}_d = \dot{o}_a + Rz(\alpha - \theta - \beta_1 - \beta_2)\Omega_z(\alpha' - \theta' - \beta_1' - \beta_2')$$

where  $V_{a1}$  is the vehicle reversing velocity along  $a1$  direction and  $\Omega_z(\theta) = Rz(\theta)^T R'_z(\theta)$

Velocities of the origins of each frame in their 1,2,3 directions. For the industrial scale prototype lengths in centimetres are,  $L_0 = 122, L_1 = 32, L_2 = 74, L_3 = 0, L_4 = 106$ . Then,

$$A = \begin{bmatrix} 0.0135 & 0 \\ -0.0135 & 0.0094 \end{bmatrix} \quad B = \begin{bmatrix} -0.0117 \\ 0.0035 \end{bmatrix}$$

Eigenvalues of matrix  $A$  are  $\lambda_1 = 0.0094$  and  $\lambda_2 = 0.0135$ . Since eigenvalues are in the right half plane, the system is unstable. We stabilize the system using state feedback

$$\theta = -K \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

C. Stabilizing the system with controller No slip condition gives the following constraints, where  $V_{b2}, V_{c2}$  and  $V_{d2}$  are the velocities of relevant frames along direction 2.

$$V_{b2} = 0 \quad (1)$$

$$V_{c2} = 0 \quad (2)$$

$$V_{d2} = 0 \quad (3)$$

From 1,

$$\dot{\alpha} = \frac{L_0 \dot{\theta} - V_{a1} \sin(\theta)}{L_0}$$

From 2 and 3,

$$\dot{\beta}_1 = -\frac{L_2 V_{a1} \sin \theta - L_0 V_{a1} \sin \beta_1 \cos \theta + L_1 V_{a1} \cos \beta_1 \sin \theta}{L_0 L_2}$$

$$\dot{\beta}_2 = -\frac{F}{(L_0 L_4)}$$

where

$$K = [-6.7730 \ 6.3263] \text{ places the poles at } P_1 = -0.001 \text{ and } P_2 = -0.078.$$

### III. SIMULATION RESULTS

#### A. Stabilizing the system from a given orientation

The simulation results correspond to the industrial scale prototype. Initial conditions for the prototype were taken as, tions with respect to earth frame are given by

$$V_b = R_z(\alpha - \theta)^T o'_b$$

$$V_c = R_z(\alpha - \theta - \beta_1)^T o'_c \quad V_d = R_z(\alpha - \theta - \beta_1 - \beta_2)^T o'_d$$

Fig.2 demonstrates a case where the trailer system reverses along a straight line while stabilizing the pivot angles.

$$\begin{aligned} x_{(0)} &= 0 & y_{(0)} &= 0 & \alpha_{(0)} &= 1.2 & \dot{\theta}_{(0)} &= 0 \\ \beta_{1(0)} &= 0.2 & \beta_{2(0)} &= -0.2 & \dot{\alpha}_{(0)} &= 0 & \dot{\beta}_{1(0)} &= 0 \\ \dot{\beta}_{2(0)} &= 0 & \theta_{(0)} &= -K \begin{bmatrix} \beta_{1(0)} \\ \beta_{2(0)} \end{bmatrix} &= 0. \end{aligned}$$

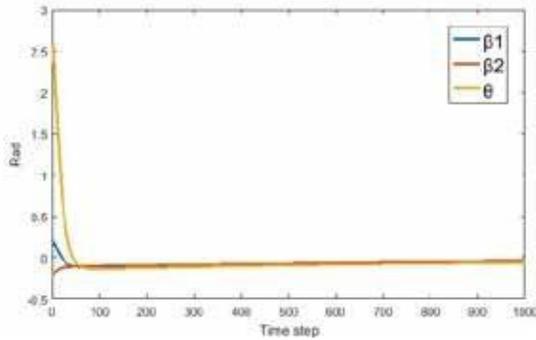


Fig. 2. Variation of pivot angles with steering angle when stabilizing

#### B. Taking curved paths

To stabilize the system while introducing small curvatures, the controller equation can be modified by adding a small bias  $d$  constrained by the small angle approximation.

$$\theta = -K \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + d$$

Fig.3 shows the simulation results for taking a curvature path starting from the same initial position. According to the results, it can be seen that the system stabilizes while maintaining pivot angles at a constant value. The relevant curved path of the vehicle can be observed in Fig.4.

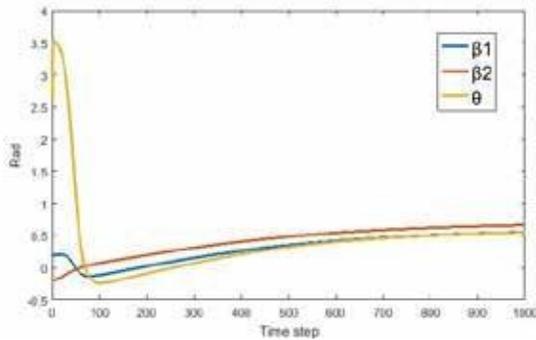


Fig. 3. Variation of pivot angles with steering angle when taking a curvature

#### IV. HARDWARE IMPLEMENTATION

##### A. Fully automated prototype

A hardware prototype was implemented with two passive trailers and a differential driven engine part that could be actuated using continuous servo motors. To measure the pivot angles potentiometers were used and the controller was implemented on an arduino board.

The first task was to stabilize the system. To achieve this, relevant gains were obtained as described in the mathematical model. Then the system was able to reverse on a straight line

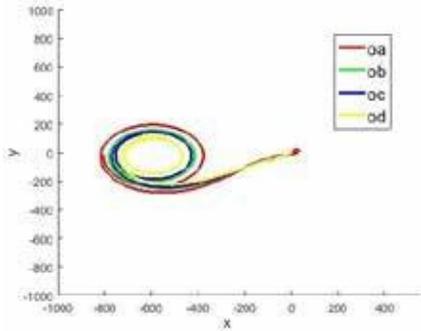


Fig. 4. Vehicle movement when taking a curvature



Fig. 5. Fully automated prototype

while maintaining the pivot angles at zero. Next task was to reverse in a curved path. For this a small bias was introduced as described in section III B. With the bias, the system was able to successfully reverse in a curved path while stabilizing the pivot angles. After successfully stabilizing the system and reversing in curved paths, reversing the system along given path was tested. For that visual sensing was used with a camera. The desired path was marked using a coloured strip and the system should reverse along that path. To identify the path using the camera, another path detection algorithm was implemented using OpenCV as described in Fig.6. The image processing was done on a Raspberry Pi computer. Processed image data was passed onto the controller on an arduino system via serial connection. A video recording of the system performance is available at <https://youtu.be/Px5YawUA4k8>.

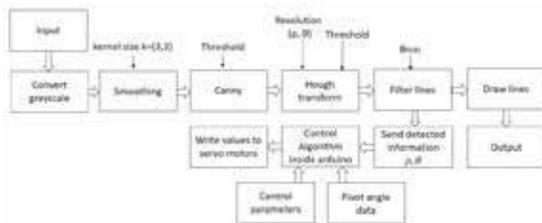


Fig. 6. Line detection algorithm

##### B. Industrial scale prototype

Next an industrial scale prototype was developed as shown in Fig.7. This prototype has one trailer and two pivot joints.



Fig. 7. Industrial scale prototype

Hence it is equivalent to the system described in Fig.1. The simulation results were generated considering this industrial scale prototype.



Fig. 8. Display

This prototype is a manual system where a human driver has to do the steering. There are no actuators available in this vehicle yet. According to the pivot angles, desired steering angle is calculated inside the controller. Pivot angles were measured using potentiometers installed at the pivot joints. Both desired and actual steering angles are displayed to the driver as in Fig.8 where the driver has to track the desired steering angle manually while reversing. When it is required to take curved paths driver can introduce the  $d$  term to the controller by rotating a knob. Since prototype was heavier than expected, initial testing was done manually, pushing the vehicle while steering. Video recording of the prototype is available at [https://youtu.be/n7IWPR3\\_g-E](https://youtu.be/n7IWPR3_g-E).

#### V. CONCLUSIONS

This paper presents a non-linear controller for reversing a truck and trailer mechanism. The controller is verified using simulations as well as hardware prototypes. These prototypes could reverse along a straight line or a curve under the small angle